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DETERMINATION OF SYNTHETIC FLOOD HYDROGRAPH IN UNGAUGED CATCHMENTS

Summary

A method for determining synthetic flood hydrograph for small ungauged catchments is presented in the paper. The model uses the SCS excess rainfall model with daily precipitation of 1% exceedance probability as the input data. The excess rainfall is transformed to surface runoff by the geomorphological runoff model.

Key words: synthetic hydrograph, SCS, unit hydrograph

INTRODUCTION

The synthetic flood hydrograph is a theoretical typical hydrograph describing a flood that can occur under certain conditions for a given peak flow [Ozga-Zielińska, Brzeziński, 1997]. The use of synthetic hydrographs is recently of growing interest in design, both in water management and hydraulic construction designing. This is due to the fact that in comparison to a design flood value, synthetic hydrograph greatly increases the amount of information adding such quantties like the volume of the flood wave and its time course. For this reason, synthetic hydrograph may be helpful in the design of reservoirs, embankments and also to determine the flood zone or, in a wider range, in issues related to flood risk [Gądek, 2010].

There are few methods that allow specifying the time course a synthetic hydrograph. Some of them are shown below.

In the paper a method is presented of determining the synthetic hydrograph in the ungauged catchment based on the geomorphologic model of runoff from the catchment.

THE REITZ-KREPS METHOD

The method of Reitz-Kreps [Lambor, 1962] describes the shape of the synthetic hydrograph by two equations: one for the rising and one for the falling limbs of the outflow hydrograph.

For the rising limb of the hydrograph, that is for the time $0 \leq t \leq t_k$, the equation takes the form:

$$Q(t) = Q_{p\%} \sin^2\left(\frac{\pi}{2}\frac{t}{t_k}\right) \tag{1}$$

The falling limb of the outflow hydrograph, for time $t > t_k$, is described by the equation:

$$Q(t) = Q_{p\%} e^{-\alpha(t-t_k)} \tag{2}$$

where:

Q – flow at time t [m^3/s],

$Q_{p\%}$ – peak flow of p% exceedance probability, [m^3/s],

t_k – rising time of the outflow hydrograph, [h],

α – coefficient determined on the basis of historical runoff hydrographs, [1/h].

The value of α can be determined from the equation:

$$\alpha = \frac{\ln 2}{(s-1)t_k} \tag{3}$$

where s is the flood hydrograph slenderness coefficient [-] defined as

$$s = \frac{t_{0.5Q_{\max}}}{t_k} \tag{4}$$

where $t_{0.5Q_{\max}}$ is the time [h] from the beginning of the hydrograph to the time instant when the flow at the falling limb is equal to a half of the maximum flow $Q_{\max}$ (=$Q_{p\%}$).

THE WARSAW UNIVERSITY OF TECHNOLOGY METHOD

This method [Hydroprojekt, 1971] requires selecting a minimum of six hydrographs of the largest unimodal floods from a set of observations covering the period for which the probable flows were calculated. For each hydrograph a base flow Q_0 (the flow at the moment the hydrograph begins to rise) is determined; it is a starting point of the flood. The ending time of the flood hydrograph is defined as the time on the falling limb when the flow equals to Q_0. The rising limb time t_k is the next value that should be determined for each hydrograph.

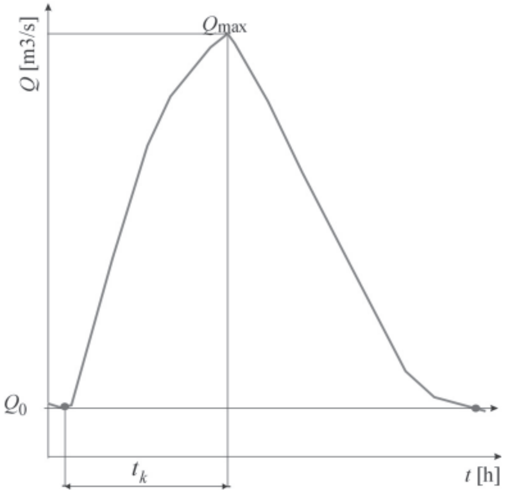

Figure 1. Flood hydrograph parameters

After determining these values, it is necessary for each j-th hydrograph, Q_j, $j = 1, 2, \ldots, n$, to read the flow values Q_{j,t_i} at the fixed moments of time t_i, $i = 1, 2, \ldots, 21$, equal to $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 1.0, 1.05, 1.1, 1.4, 1.6, 1.8, 2.0, 3.0, 3.5, 4.0, 5.0\} \times t_k$,.

In the next step the values S_i of the standardized outflow hydrograph are calculated:

$$S_i = \frac{1}{n} \sum_{j=1}^{n} \frac{Q_{j,t_i}}{Q_{j\,\max}} \qquad (5)$$

where $Q_{j\,\max}$ is the maximum flow of the j-th real hydrograph, [m^3/s].

The values of the synthetic flood hydrographs are obtained by multiplying the values of the standardized hydrograph by the value of the flow of a certain exceedance probability.

THE PROPOSED METHOD OF DETERMINING SYNTHETIC FLOOD HYDROGRAPH

The methods for determining synthetic flood hydrographs, presented above, can be applied only in the gauged catchments of any area. Most of the issues for which there is a need to calculate the synthetic flood hydrographs with a peak flow of the given exceedance probability concerns the ungauged catchments, especially those of area of up to 50 km^2. In the Department of Hydrology of Cracow University of Technology a method was proposed of determining such hydrograph using mathematical modeling. The method uses widely applied in our country the SCS effective rainfall model and the model of effective rainfall transformation into surface runoff using geomorphological mode of catchment runoff.

The SCS effective rainfall model

The basic assumption of the method [Banasik, 1994] is that the ratio of the height H of effective rainfall to the total precipitation P reduced by the initial loss (initial abstraction) I_a is equal to the ratio of actual infiltration F to the maximum potential storage S of the catchment, what can be expressed by the following equation:

$$\frac{H}{P-I_a}=\frac{F}{S} \tag{6}$$

or, after transformation:

$$H=\frac{(P-I_a)^2}{P-I_a+S} \tag{7}$$

The amount of effective precipitation $H(t)$ summed over the time interval from 0 (start of rainfall) to time t is:

$$H(t)=\begin{cases}0 & \text{when} \quad P(t)-0.2S\leq 0\\ \dfrac{\left(P(t)-0.2S\right)^2}{P(t)+0.8S} & \text{when} \quad P(t)-0.2S>0\end{cases} \tag{8}$$

where:

P – average rainfall summed over the time interval from 0 to time t, [mm],

S – maximum potential storage of the catchment [mm] depending on the CN number according to the formula:

$$S=25.4\left(\frac{1000}{CN}-10\right) \tag{9}$$

Initial losses and other losses (time-varying) are taken into account through CN, the Curve Number. It is determined on the basis of the permeability of soils in the catchment, land use, crop type and soil moisture in the catchment in the period preceding the analyzed precipitation.

Depending on the type of formation of surface runoff model, four groups of soils are developed. Under Polish conditions this classification is difficult to use. Such a classification for Polish conditions and types of soils given by the Polish Soil-Science Society and based on the literature information on the mechanical composition of soils and values of infiltration rates has been developed by Ignar [1988]. The classification allows application of agricultural soil maps developed for the Polish territory to the SCS model.

The CN values refer to the average moisture conditions [Banasik, 1994].

Geomorphological model of runoff

Catchment geomorphology is a key factor in formation of the outflow from a catchment. Water from rainfall flows first over the ground surface reaching then watercourse and is transformed along into the cross-section closing the catchment. Using these processes, Rodriguez-Iturbe and Valdes [1979] developed a theory of geomorphological unit hydrograph.

Calculation of the parameters of the triangular unit hydrograph from the equation of the geomorphological instantaneous unit hydrograph is very difficult. Rodriguez-Iturbe and Valdes gave the formulas for their calculation:

a) peak flow height, q_p, [mm/h]:

$$q_p = 1.31 R_L^{0.43} \frac{v_n}{L_0} \quad (10)$$

where:

R_L – stream length ratio, [-]

v_n – flow velocity, [m/s],

L_0 – length of the main stream, [km].

The flow velocity can be calculated from the equation:

$$v_n = 0.665\, \alpha_n^{0.6} \left(0.1 H_{ef}\, A_w\right)^{0.4} \tag{11}$$

where:

α_n – kinematic wave coefficient,

H_{ef} – intensity of effective rainfall, [mm/h],

A_w – catchment area, [km^2].

Kinematic wave coefficient can be calculated from the formula:

$$\alpha_n = \frac{S_0^{1/2}}{n_M\, b^{2/3}} \tag{12}$$

where:

S_0 – slope of the main stream bed, [-],
b – equivalent width of the stream bed, [m],
n_M – Manning roughness coefficient.

b) time t_p of peak flow occurrence, [h]:

$$t_p = 0.44\left(\frac{R_B}{R_A}\right)^{0.55} R_L^{-0.38}\,\frac{L_o}{v_n} \tag{13}$$

where:

R_A – catchment area ratio, [-],
R_B – bifurcation ratio, [-].

Application of the geomorphological model to determine the synthetic hydrograph

Determining the course of a synthetic flood hydrograph is made in two independent stages. In the first stage the flood hydrograph is determined that would be created when a given hourly distribution of daily rainfall of a preset exceedance probability occurs. In the presented method it is assumed that the volume of the synthetic flood hydrograph of a given exceedance probability of the peak flow is equal to the volume of the effective precipitation resulting from the rainfall of the same exceedance probability. The calculations in the catchment area are carried out using the parameters determined basing on the type of soils, land use land, the amount of antecedent rainfall and river network. Distribution of hourly daily precipitation with a given exceedance probability is arbitrary and is defined by the user individually.

In the second stage the synthetic flood hydrograph is developed for a given peak value and the values determined in the first stage, i.e., the flood volume and the times of rising and falling limbs of the hydrograph. The flow height is defined as the peak flow of a given value of exceedance probability, e.g., $Q_{p=1\%}$.

Numerical experiment

Determination of synthetic hydrographs is based on the SCS effective rainfall model and geomorphologic model of catchment runoff [Więzik and Banach, 1990; Więzik, 1988]. It was assumed that the calculations of synthetic hydrograph of outflow from the catchment will be made through optimization of selected parameters of the geomorphological model. To this purpose, the "golden ratio" optimization method was used [Findeinsen, 1970]. The form of the criterion function is adapted in this way as to get the best fit of the peak flow. The function was defined by the following formula:

$$F_c = \min \left(Q_{\max o} - Q_{p\%} \right)^2 \tag{14}$$

where:

F_c – the value of the criterion function,

$Q_{\max,o}$ – peak flow of the calculated synthetic hydrograph (from optimization), [m^3/s],

$Q_{p\%}$ – flow of the p% exceedance probability, [m^3/s].

Exemplary calculations were made at selected cross-sections of two rivers: Prądnik and Żylica in southern Poland. The catchments of these rivers are different: Żylica is a mountain catchment, while the Prądnik catchment is upland.

Table 1. Selected physiographic parameters of the catchments of the Żylica and Prądnik rivers

Parameter	Prądnik catchment	Żylica catchment
Catchment area [km^2]	67.5	52.256
Length of the main stream L_0 [km]	11.54	17.693
Longitudinal slope of the highest order stream S_0 [-]	0.0626	0.0187
Horton-Strahler ratios:		
Bifurcation ratio R_b [-]	1.140	7.000
Stream length ratio R_l [-]	0.700	0.373
Stream area ratio R_a [-]	0.701	1.567

The considered method of determining synthetic hydrograph consisted in transforming, according to the base time t_b, the standardized unit runoff hydrographs of a given distribution of daily rainfall with a given exceedance probability. It was assumed that the base times of the individual hydrographs will be multiplied by the same value, so that after the superposition of all elementary hydrographs the best fit of the calculated peak flow to the peak flow of a given exceedance probability would be obtained.

The unit hydrograph volume is assumed to be constant (Fig. 2):

$$\frac{1}{2} q_p t_b = \frac{1}{2} q_{po} t_{bo} \tag{15}$$

where:

q_p – peak flow of the calculated unit hydrograph, [mm],
t_b – base time of the calculated unit hydrograph, [min],
q_{po} – peak flow of the transformed unit hydrograph, [mm],
t_{bo} – base time of the transformed unit hydrograph, [min],

$$t_{bo} = t_b \, mn \tag{16}$$

mn – multiplier [-].

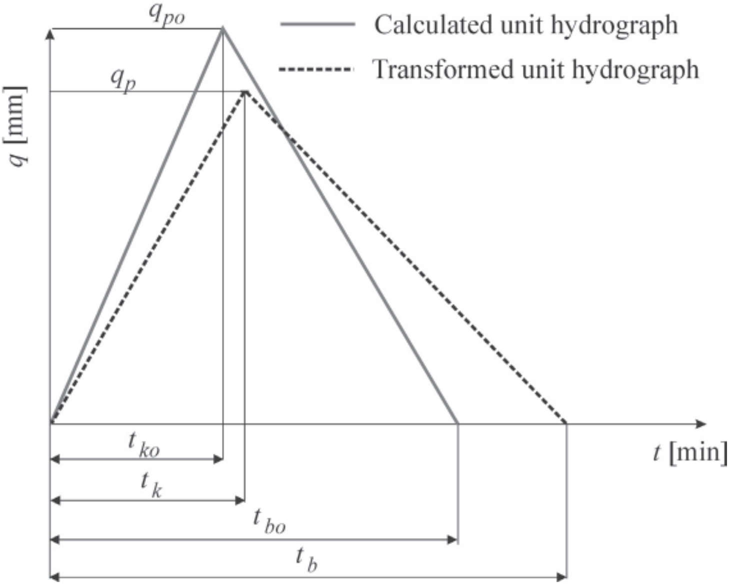

Figure 2. Unit hydrograph

Figures 3 and 4 present the obtained results for 1% peak flows of 85.0 m^3/s for the Prądnik river and 60.0 m^3/s for the Żylica river. The Q_{calc} hydrograph is calculated for hourly distribution of a given daily rainfall of the given exceedance probability. The graph labeled "Optimisation of tb" represents the synthetic hydrograph for which the multiplier of t_b was optimized.

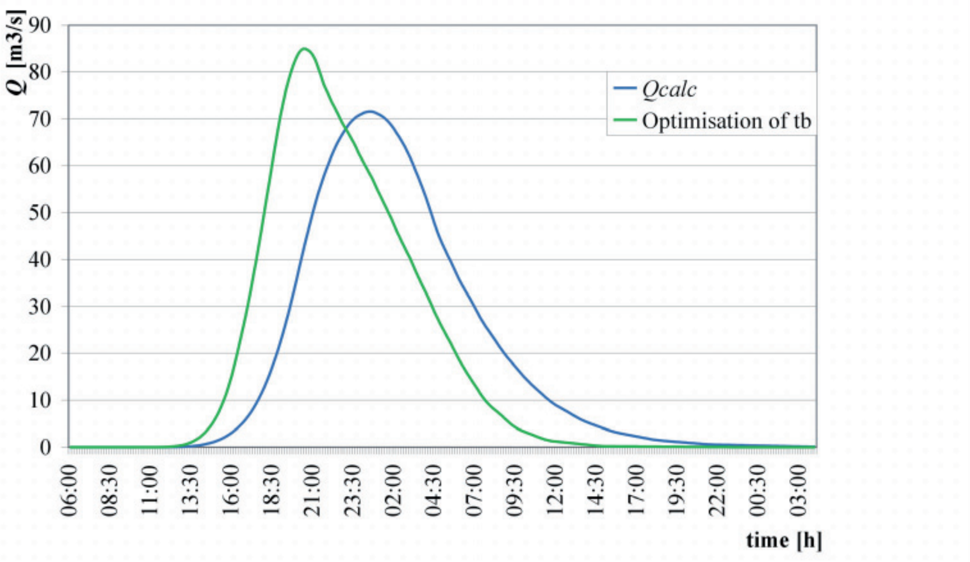

Figure 3. Calculated and synthetic flood hydrographs wave for the Prądnik river catchment

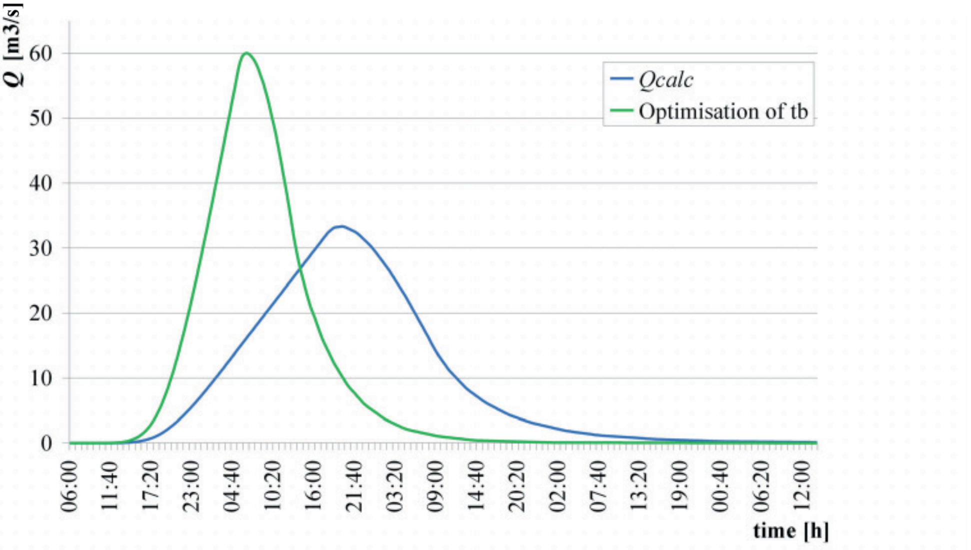

Figure 4. Calculated and synthetic flood hydrographs wave for the for Żylica river catchment

CONCLUSION

The proposed method for determining the synthetic flood hydrographs of a given peak flow is designed for small ungauged catchments. Basing on the obtained results a conclusion can be drawn that the synthetic hydrograph obtained by optimizing the base time multiplier achieves the required peak flow, i.e., $Q_{max,o} = Q_{p\%}$, also the flood hydrograph volume is preserved. Therefore, the proposed method of optimizing the base time multiplier can be regarded as a proper method of calculating the synthetic flood hydrographs in ungauged catchments.

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