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THE EFFICIENCY OF SOME FORECASTING METHODS APPLIED TO ANNUAL MINIMUM FLOW SERIES

Summary

Four methods of forecasting: „no-change”, LOESS, local linear regression and Holt-Winters were applied to annual minimum water levels observed at ten cross-sections of two tributaries of the Vistula river. The 1-, 2-, …, 5-year forecasts were made for each year after some initial year, and four quality measures: bias, root mean square error, mean absolute error and maximum absolute error were calculated for each time series and lead time. The naïve model turned out to be always the worst in it bias and almost always very good, sometimes the best regarding the other measures.

Key words: forecasting, LOESS, local linear regression model, Holt-Winters model

INTRODUCTION

In many forecasting problems situations exist where nonlinearity is seen and, additionally, of alternating monotonicity, sometimes quickly. In such cases and when no physical model is available, global approach, e.g., nonlinear regression, in finding trend and, basing on it, making forecasts, is not justified. Using a nonparametric, or local, method may be the solution. There are several methods of nonparametric approach [Helsel and Hirsch, 1997]; some of them are tested in the paper.

Annual minimum levels of a river are of interest for their connection, among others, with the elevation of river bottom (erosion problems) and water supply problems. Of interest, therefore, is predicting the future values of annual minimum levels of and the related uncertainty.

The aim of the paper is to study the efficiency of four forecasting methods in their application to annual minimum level time series. The methods selected are: „no-change” or naïve, LOESS, local linear regression and Holt-Winters models. Four forecast efficiency measures are used: bias, *B*, root mean square error, *RMSE*, mean absolute error, *MAE*, and maximum absolute error, *mxAE*.

METHODS

For given time series y_t, $t = 1, 2, \ldots, n$, of annual minimum flows a forecast f_t is sought for all time instants $t > t_0$. Forecasting is made using the following four models.

The “naïve” or “no-change” model. The assumptions behind this model are as simple as possible and can be expressed by the following formula:

$$f_{t+i} = y_t, \quad i = 1, 2, \ldots, m \tag{1}$$

where f_{t+i} is the forecast of y for time $t + i$ and m is the maximum lead time of the forecast. In the paper the model serves as the reference in assessing the forecasting quality and usability of other models.

The LOESS model. There exist several nonparametric regression methods of which a method called LOWESS or LOESS (locally weighted scatterplot smoothing) [Cleveland i Loader, 1995] turned out to be very useful. In the paper, the method is applied in its linear version.

Assume that the mean annual minimum level $y(t)$ of a river at time instant t in the neighbourhood of time instant t_i can be described by a linear formula

$$y(t) = a + b(t_i - t) + \varepsilon(t) \tag{2}$$

where ε is normal random error, N(0,σ). Regression coefficients a and b are calculated (separately for each t) by the method of locally weighted least squares with weights w:

$$(a,b) = \arg\min \sum_{i=1}^{p} w\left(\frac{t_i - t}{h}\right) e_i^2 = \arg\min \sum_{i=1}^{p} w\left(\frac{t_i - t}{h}\right) \left(y_i - a - b(t_i - t)\right)^2 \tag{3}$$

After finding the regression coefficients a and b, we get from (2): $y(t_i) = a$. Parameter h, known also as the bandwidth, decides of the amount of smoothing of the cloud of points (t_i, y_i). The greater the value of h, the smoother (less variable) is the formed line. It should be noted that the regression coefficients are calculated for each point t at which the estimated y value is to be found, and therefore the straight line (2) is valid only at one point t. The trend found by a nonparametric estimation method is not expressed explicitly in the form of equation – it is almost exclusively expressed in a graphic form.

Many formulas can be used for weight function $w(z)$. In practice, frequently used function is a tri-cube function [NIST, 2011], which was used in the paper in a one-sided form (Fig. 1):

$$w(z) = \begin{cases} \left(1 - |z|^3\right)^3 & -1 < z \leq 0 \\ 0 & z \leq -1 \text{ or } z > 0 \end{cases} \tag{4}$$

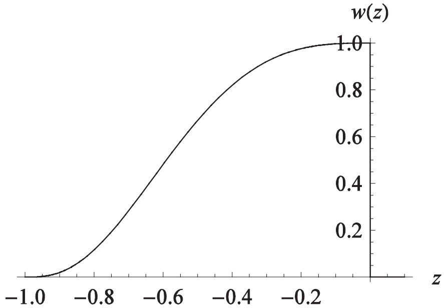

Figure 1. The LOESS weight function (4) used in the paper.

where

$$z = \frac{t_i - t}{h} \tag{5}$$

The bandwidth h is by definition the length of an assumed fixed instant of time (5-year period was adopted). Equations (4) and (5) show that the forecasted value will be affected by the values from the period $t_i - h + 1$, $t_i - h + 2$, …, t_i with their influence decreasing with increasing temporal distance from the time t_i.

Generally speaking, the choice of weight function $w(z)$ and the bandwidth h is a subjective element of the non-parametric estimation, although there are some reasonable theoretical and practical reasons for their choice.

The LOESS forecast for time $t + i$ is

$$f_{t+i} = a + b(t + i) \tag{6}$$

[Li and Heckman, 1996] where a and b are obtained by method (3) applied to the last p observations before $t + 1$: $\{y_{t-p+1}, y_{t-p+2}, \ldots, y_t\}$.

Local linear regression model. Because of the expected nonlinearities of annual minimum flow time series, it was assumed that only q last observations before $t + 1$ are of value for forecasting m future values. The linear regression model is therefore local and the forecast is calculated as

$$f_{t+i} = a + b(t + i) \tag{7}$$

where the regression parameters, *a* and *b*, are obtained by minimizing least squares for given series of observations $\{y_{t-q+1}, y_{t-q+2},\ldots, y_t\}$ with weights equal to one as is for usual regression. In the paper the value $q = 5$ was adopted.

Holt-Winters forecasting model. The Holt-Winters forecasting model used in the paper can be summarized in the following equations [NIST, 2011]:

$$f_{t+i} = A_t + iB_t \tag{8}$$

$$A_t = \alpha y_t + (1-\alpha)A_{t-1}, \quad t > 1 \tag{9}$$

$$B_t = \beta(A_t - A_{t-1}) + (1-\beta)B_{t-1}, \quad t > 1 \tag{10}$$

with initial values:

$$A_1 = y_1; \quad B_1 = y_2 - y_1 \tag{11}$$

The two parameters of the model, α and β, are found by minimizing the model mean square error for series of observations $\{y_2, y_3,\ldots, y_t\}$.

MEASURES OF FORECAST UNCERTAINTY

Four measures of model forecast uncertainty were used: bias *B*, root mean square error *RMSE*, mean absolute error *MAE*, and maximum absolute error *MxAE*. They are defined by the following equations:

– bias:

$$B = \frac{1}{n-m-t_0} \sum_{t=t_0+1}^{n-m} \left(f_t - y_t \right) \tag{12}$$

– root mean square error:

$$RMSE = \sqrt{\frac{1}{n-m-t_0} \sum_{t=t_0+1}^{n-m} \left(f_t - y_t \right)^2} \tag{13}$$

– mean absolute error:

$$MAE = \frac{1}{n-m-t_0} \sum_{t=t_0+1}^{n-m} | f_t - y_t | \tag{14}$$

– maximum absolute error

$$mxAE = \max_{t=t_0+1,\ldots,n-m} | f_t - y_t | \tag{15}$$

Although different initial time instant t_0 could be assumed, to make the comparison of efficiency measures adequate, a single value of $t_0 = 5$ was used throughout the paper.

DATA

Ten non-interrupted time series of length at least 50 years were selected to the analysis. The cross-sections were selected on two right-side tributaries of the Vistula river and were as follows: Mszana Dolna, Stróża, Gdów and Proszówki on the Raba river and Kowaniec, Krościenko, Nowy Sącz, Czchów, Zgłobice and Żabno on the Dunajec river. Time course of the levels is shown in Fig. 2.

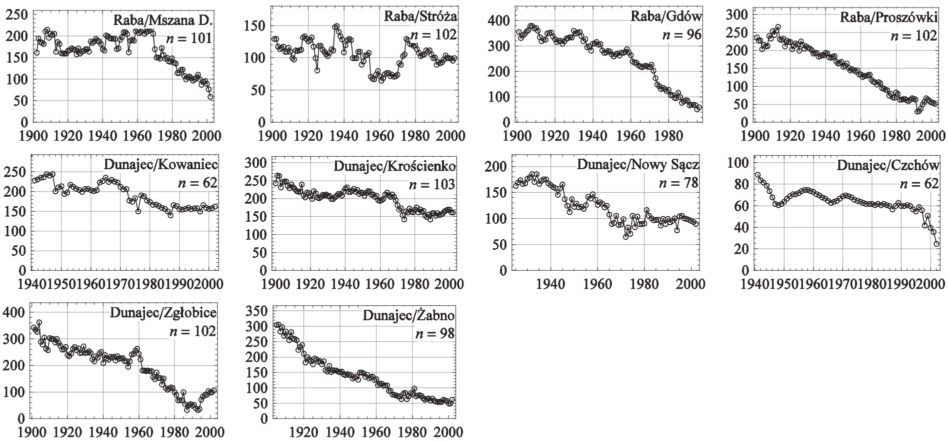

Figure 2. Annual minimum level time series at the selected cross-sections

RESULTS AND DISCUSSION

For each of 10 time series four forecast models were applied. It was assumed that a 5 year-period preceding the forecast is sufficient for LOESS and linear regression and 1-, 2-, …, 5-year forecasts were made for each year $t > 5$. Parameters of the Holt-Winters model were calculated by minimizing *RMSE* for each $t > 5$, basing on series $y_1,\dots,y_t$. Exemplary graphical visualizations of 1-, 2-, …, 5-year forecast produced by each method is shown in Fig. 3.

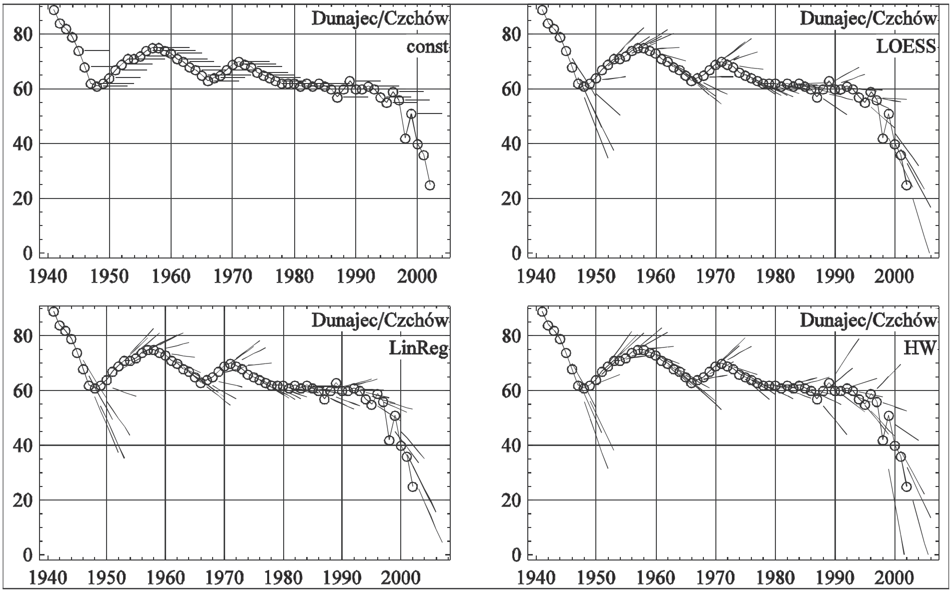

Figure 3. Five-year forecasts by four forecasting methods:
const, LOESS, linear regression (LinReg) and Holt-Winters (HW)
at the Czchów cross-sections on the Dunajec river

Forecast lines in Fig. 3 illustrate the forecasting ability of the applied methods. LOESS, LinReg and HW are acceptable in these parts of a time series which are of monotonic character. Points at which monotonicity alternates and their right-hand neighbourhood generate the greatest errors. The naïve model shows here higher robustness.

To make comparison of the for methods as adequate as possible, forecasting values of all methods were compared with the observed ones for all time instants $t = 6, \ldots, n - 5$ ($m = 5$), and using this data four quality measures, i.e., *B, RMSE*, *MAE* and *mxAE* (equations (12), (13), (14) and (15), respectively), were calculated for each of 10 cross-sections and each lead time equal to 1, 2, …, 5 years. The results are illustrated in Fig. 4; full information is given in Table 1.

Table 1. Values of the applied forecasting efficiency measures for lead times 1, 2, …, 5 years. The values greater than 9.9 cm were rounded to whole numbers

	Raba/Mszana D.					**Raba/Stróża**					**Raba/Gdów**					**Raba/Proszówki**				
***B*, cm**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**
const	1.3	2.7	4.1	5.2	6.5	0.2	0.4	0.5	0.6	0.8	3.3	6.8	10.3	14	17.6	1.7	3.3	5	7	8.9
LOESS	0.3	0.7	1.1	1.2	1.8	-0.1	-0.1	-0.1	-0.3	-0.5	0.3	0.7	1.2	1.9	2.7	-0.1	-0.1	0	0.2	0
LinReg	0.4	0.8	1.2	1.5	2.1	-0.1	-0.1	-0.3	-0.5	-0.7	0.3	0.7	1.3	2	2.7	-0.1	-0.1	-0.2	-0.2	-0.5
HW	0.2	0.6	1	1.2	1.5	-0.4	-0.5	-0.8	-1.1	-1.4	0.1	0.5	0.9	1.4	2	-0.4	-0.5	-0.6	-0.4	-0.4
***RMSE*, cm**																				
const	13	17	19	21	23	10	15	17	18	19	12	18	24	28	32	10	13	15	17	19
LOESS	15	23	29	36	41	13	21	26	31	36	14	23	32	41	51	11	16	21	26	30
LinReg	16	22	28	34	39	14	20	25	29	34	15	24	33	42	50	11	16	21	25	29
HW	15	22	27	32	38	11	16	20	22	24	14	22	32	42	53	11	14	18	23	27
***MAE*, cm**																				
const	9	13	14	16	17	7	11	13	14	15	9	14	19	23	26	7	10	12	14	15
LOESS	11	18	22	28	32	10	16	21	24	28	12	18	26	34	43	8	12	15	19	22
LinReg	12	17	21	26	30	11	16	20	24	27	13	19	27	35	42	8	12	15	18	21
HW	10	17	21	25	29	8	12	15	17	19	11	18	26	33	42	8	11	14	16	19
***mxAE*, cm**																				
const	42	55	65	65	68	38	40	51	47	56	32	54	78	85	96	34	36	41	54	44
LOESS	48	65	86	91	108	52	67	77	100	103	37	54	80	98	117	38	49	68	91	100
LinReg	51	70	77	80	101	45	58	70	88	89	39	58	76	88	106	39	47	65	81	88
HW	45	57	70	96	116	50	64	71	82	91	37	58	86	106	129	37	44	60	75	90

	Dunajec/Kowaniec					**Dunajec/Krościenko**					**Dunajec/Nowy Sącz**					**Dunajec/Czchów**				
***B*, cm**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**
const	1.4	2.9	4.6	5.4	6.4	0.9	1.8	2.6	3.3	4	1.1	2.2	3.5	4.8	5.9	0.9	1.4	1.9	2.1	2.5
LOESS	0.1	0.3	0.5	-0.3	-1	-0.1	-0.1	-0.2	-0.6	-1.1	0.1	0.2	0.5	0.9	0.9	0.2	0.2	0.1	0	0.1
LinReg	0.2	0.3	0.5	-0.2	-0.8	-0.1	-0.1	-0.3	-0.7	-1.2	0.1	0.3	0.6	1	0.7	0.2	0.2	0.1	0	-0.1
HW	-0.4	-0.8	-1.1	-2.3	-3.4	0	0	-0.1	-0.4	-0.9	0	0.4	0.8	1.2	1.3	0.1	0	-0.2	-0.5	0.3
***RMSE*, cm**																				
const	13	15	17	19	21	9	11	12	14	15	13	13	16	16	18	4	5	6	7	8
LOESS	14	18	23	30	35	10	14	17	20	26	14	17	22	25	29	4	5	7	9	12
LinReg	14	18	23	29	34	10	13	16	20	24	13	17	21	23	25	4	5	7	9	12
HW	14	18	22	28	35	10	14	16	19	23	13	14	18	20	24	4	6	8	10	12
***MAE*, cm**																				
const	8	12	13	14	16	7	9	10	11	12	10	10	12	13	15	2	3	4	5	6
LOESS	10	14	19	24	29	8	11	13	16	20	10	14	18	20	22	2	3	5	6	8
LinReg	10	14	18	24	28	8	10	12	16	20	10	13	17	18	20	2	4	5	7	9
HW	9	13	16	20	24	7	11	13	15	18	10	12	15	16	19	2	4	5	7	9
***mxAE*, cm**																				
const	44	40	44	57	56	30	36	36	42	53	35	41	53	45	43	14	17	26	20	31
LOESS	46	50	55	64	72	31	40	60	62	71	42	44	63	71	87	15	14	21	27	34
LinReg	46	53	53	60	69	29	35	56	56	65	40	37	55	61	75	14	14	22	29	36
HW	49	54	62	92	139	34	46	59	60	69	38	36	46	45	59	20	21	29	30	38

	Dunajec/Zgłobice					Dunajec/Żabno				
***B*, cm**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**	***t*+1**	***t*+2**	***t*+3**	***t*+4**	***t*+5**
const	1.9	3.8	6	7.7	9.6	2.2	4.8	7.3	9.5	12.1
LOESS	-0.5	-1.2	-1.8	-3	-4.4	-0.4	-0.3	-0.5	-1	-1.3
LinReg	-0.5	-1.2	-1.8	-3.1	-4.8	-0.4	-0.4	-0.6	-1.1	-1.5
HW	-2.4	-3.9	-5.1	-7	-8.7	-0.9	-1.3	-2	-2.8	-3.4
***RMSE*, cm**										
const	17	22	25	28	31	11	14	14	18	20
LOESS	20	29	37	46	54	12	15	16	21	25
LinReg	20	29	37	44	51	12	13	16	20	23
HW	18	23	27	30	33	11	12	14	18	20
***MAE*, cm**										
const	13	17	19	23	25	8	10	11	13	15
LOESS	16	23	30	36	43	9	11	13	16	18
LinReg	16	23	29	34	40	9	11	12	15	17
HW	14	18	20	22	25	8	9	11	14	15
***mxAE*, cm**										
const	46	67	82	82	84	32	57	51	58	73
LOESS	56	72	110	125	138	32	45	48	57	90
LinReg	51	78	117	138	152	33	36	42	57	68
HW	57	73	82	84	91	31	37	39	47	54

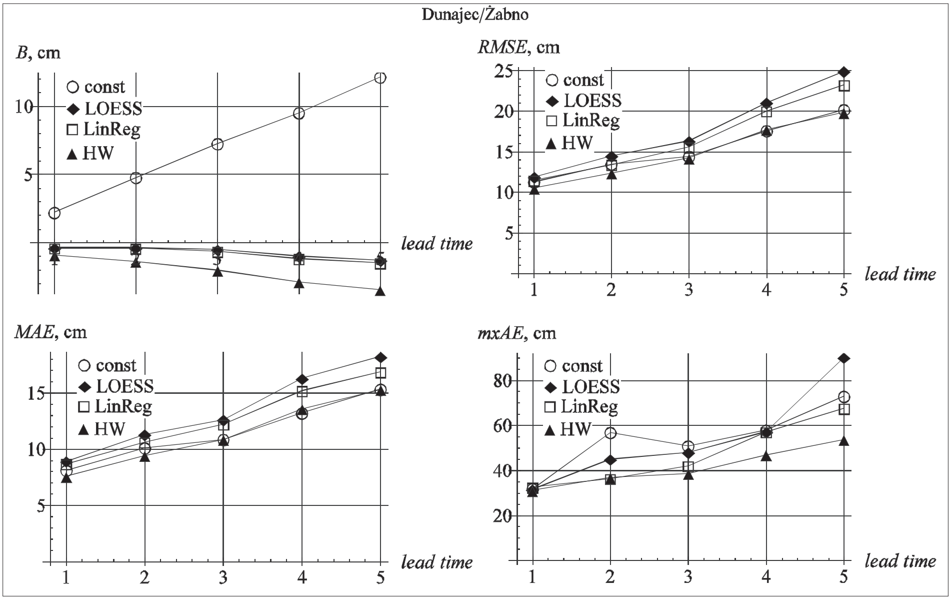

Figure 4. Rather typical image of the relationship between theforecast efficiency measures and lead time and the applied forecasting methods shown for the Żabno cross-sections on the Dunajec river

What is striking when looking at Fig. 3 is the big discrepancy between the bias of the naïve model and the biases of other models. The data in Table 1 confirm this finding, although this discrepancy in absolute values is in some case a few centimeters only. As regards the rest three measures: *RMSE*, *MAE* and *mxAE*, the naïve model is almost always the best of the three methods. Only if time series is roughly monotonic, as in the case of Dunajec/Żabno (see Fig. 1), the HW model seems to be better in *RMSE*, *MAE* and *mxAE* than the naïve model.

It should be underlined that no rules in excluding some data for forecasting were applied. Establishing such rules is justified in some cases, e.g., when great jump up or down occurred. Such event excludes the before-the-jump historical data; the consequence of this fact may be excluding all history and beginning forecasting as if no data were available.

Of all the three models the Holt-Winters model seems to be the best. However, haw can be seen in Table 1, the differences between the HW model efficiency characteristics and those of the LOESS and LinReg models often do not differ much. The question to be answered is whether the user should prefer the bias of a model to a measure model variability (e.g., *RMSE*) or the opposite.

CONCLUSIONS

Ten time series of annual minimum level of length exceeding 50 were used to enable comparison of forecast efficiency of 4 models to be assessed. The efficiency of the naïve, LOESS, local linear regression and Holt-Winters models was measured by means of four characteristics: bias, *B*, root mean square error, mean absolute error, and maximum absolute error. The naïve model turned out to be the worst in its bias and very good, sometimes the best, as regards the other efficiency characteristics. It seems that a careful insight in historical data usefulness for forecasting is necessary to exclude high jumps that suggest a new future, different from the future suggested by the before-the-jump data.

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